Investment Statistics: Definitions & Formulas

The following are brief descriptions and formulas for the various statistics and calculations available within the eASE Analytics system. Unless stated otherwise, calculations for periods longer than one year are annualized. For statistical significance, many regression and statistic calculations require at least 12 data points to determine results (3 years if quarterly data used or 1 year if monthly data used) and no results will be displayed (“---”) in instances where minimum data points are not available.

**Alpha** - The incremental return of a manager when the market is stationary. In other words, it is the extra return due to non-market factors. This risk-adjusted factor takes into account both the performance of the market as a whole and the volatility of the manager. A positive alpha indicates that a manager has produced returns above the expected level at that risk level, and vice versa for a negative alpha. Alpha is the Y intercept of the regression line.

\[
\text{Alpha} = \alpha = X - \text{Beta} \times Y
\]

- \( X \) = the mean return for the manager
- \( Y \) = the mean return for the index

See Also: Beta; Jensen Alpha

**Average Drawdown** – The arithmetic average of all drawdowns over a given time period.

**Batting Average** - As the name would imply, batting average is a measure of the frequency of success. This ratio is calculated by taking the number of periods where the manager equals or outperforms the selected benchmark, divided by the total number of periods. This measure indicates a manager’s frequency of success, without regard to degree of outperformance.

\[
\text{Batting Average} = \frac{\text{numOutperform}}{\text{numTotal}}
\]

- \( \text{numOutperform} \) = the number of observations where the manager outperforms the benchmark
- \( \text{numTotal} \) = the number of total observations

**Bias Ratio** – This ratio measures how far the returns of a portfolio are from an unbiased distribution.

\[
\text{Bias Ratio} = \frac{\text{Count}(r_t | r_t \in [0, \sigma])}{1 + \text{Count}(r_t | r_t \in [-\sigma, 0])}
\]

**Best Period** - The highest return for the selected time frame.

\[
\text{Best Period} = \max(R_1, R_2, \ldots, R_n)
\]

- \( R \) = the return for each period in decimal format (e.g. 5.20% = 0.052)
- \( x_i \) = the return of the data series (ith observation)

See Also: Worst Period

**Beta** - This is a measure of a portfolio’s volatility. Statistically, beta is the covariance of the portfolio in relation to the market. A beta of 1.00 implies perfect historical correlation of movement with the market. A higher beta manager will rise and fall more rapidly than the market, whereas a lower beta manager will rise and fall slower. For example, a 1.10 beta portfolio has historically been 10% more volatile than the market.
**Beta** ($\beta$) = \[
\frac{[(n)*\Sigma(x_i^*y_i)] - (\Sigma x_i)(\Sigma y_i)}{[(n)*\Sigma(y_i^2)] - (\Sigma y_i)^2}
\]

$n$ = the number of observations  
$x_i$ = the return of the first data series ($i$th observation)  
$y_i$ = the return of the second data series ($i$th observation)  
Generally, $x_i$ = the manager’s return series and $y_i$ will be a specified index (benchmark)

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**Calmar Ratio** – This ratio is calculated by dividing the annualized manager return by the max drawdown over a selected time period. This is a commonly used hedge fund measure since such funds often employ hedging strategies to protect returns in down markets; hence, the max drawdown is expected to be lower. Generally, a higher Calmar Ratio is better as it indicates the manager has higher returns and/or lower max drawdown.

\[
\text{Calmar Ratio} = \frac{\text{Annualized Manager Return}}{\text{Max Drawdown}}
\]

*See Also: Return (Annualized); Max Drawdown*

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**Correlation Coefficient** - A statistical term which defines the percent of time an index(es) or a manager(s) move in the same direction. More specifically, correlation measures the extent of linear association of two variables. Correlation coefficients can range from $-1$ to $+1$. A coefficient that is $-1$ means that the manager is perfectly negatively correlated with the index or manager against which it is regressed (move same amount in opposite directions); a coefficient of 0 signifies zero or no correlation, and finally a coefficient of $+1$ means perfect positive correlation (move the same amount in the same direction).

\[
\text{Correlation Coefficient (r)} = \frac{[(n)*\Sigma (x_i*y_i)] - (\Sigma x_i)(\Sigma y_i)}{\sqrt{[(n)*\Sigma(y_i^2)] - (\Sigma y_i)^2} \cdot \sqrt{[(n)*\Sigma(x_i^2)] - (\Sigma x_i)^2}}
\]

$n$ = the number of observations  
$x_i$ = the return of the first data series ($i$th observation)  
$y_i$ = the return of the second data series ($i$th observation)  
Generally, $x_i$ = the manager’s return series and $y_i$ will be a specified index (benchmark)

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**Correlation Matrix** - A common means of displaying the correlation results for multiple managers and benchmarks simultaneously. Reading down or across the matrix, the correlation of each manager and benchmark combination can be seen, with a diagonal separation of the matrix where the correlation results of a manager against itself are always one. A one-sided matrix shows only one iteration of each calculation (the left side of the diagonal) while a two-sided test shows each instance of the calculation results (both sides of the diagonal).

*See Also: Correlation Coefficient; Correlation of Excess Returns*

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**Correlation of Excess Returns** - Applying the same methodology as with a standard correlation coefficient calculation, the correlation of excess returns attempts to indicate linearity in the movement of manager returns above a given benchmark. This calculation is most often used to compare the excess return pattern of one manager against that of another manager and attempts to draw out greater analysis of linearity by removing the market variable both are assumed to have in common. Such analysis is useful in evaluating multiple managers within a specific style classification.

\[
\text{Correlation of Excess Returns (rex)} = \frac{[(n)*\Sigma (\text{Ex}(x_i)*(\text{Ex}(y_i))) - (\Sigma \text{Ex}(x_i))(\Sigma \text{Ex}(y_i))]}{\sqrt{[(n)*\Sigma(\text{Ex}(y_i)^2)) - (\Sigma \text{Ex}(y_i))^2} \cdot \sqrt{[(n)*\Sigma(\text{Ex}(x_i)^2)) - (\Sigma \text{Ex}(x_i))^2}}}
\]

$n$ = the number of observations  
$\text{Ex}(x_i)$ = the excess return of the first manager data series ($i$th observation)  
$\text{Ex}(y_i)$ = the excess return of the second manager data series ($i$th observation)  
*See Also: Correlation Coefficient*
**Current Drawdown** – The percentage of the most recent decline in value.

**Current Drawdown Date** – The date of the most recent decline in value.

**Downside Market Capture Ratio** - A measure of the manager's performance in down markets relative to the market itself. A value of 90 suggests the manager's loss is only nine tenths of the market's loss during the selected time period. A market is considered down if the return for the benchmark is less than zero. The Downside Capture Ratio is calculated by dividing the return of the manager during the down market periods by the return of the market during the same periods. Generally, the lower the DMC Ratio, the better (If the manager's DMC Ratio is negative, it means that during that specific time period, the manager's return for that period was actually positive).

The number of down periods for a given series \( x_1, \ldots, x_n \) is the number of negative returns in the series.

\[
DMC\ Ratio = \frac{\{ (1+R_m1)^*(1+R_m)^{1/N} \} - 1}{\{ (1+R_y1)^*(1+R_y)^{1/N} \} - 1}
\]

- \( R_m \) = return for time period when benchmark \( (R_y) \) is negative
- \( N \) = Number of years (e.g. 6 quarters = 1.5 years; 20 months = 1.667 years)

See Also:  Upside Market Capture Ratio; Down Market Return

**Down Market Return** - The annualized return for a manager during down markets, defined as periods where the return of the selected benchmark is less than zero.

\[
Down\ Market\ Return = \{ (1+R_m1)^*(1+R_m)^{1/N} \} - 1
\]

- \( R_m \) = return for time period when benchmark return is less than zero
- \( N \) = Number of years (e.g. 6 quarters = 1.5 years; 20 months = 1.667 years)

See Also: Up Market Return; Downside Market Capture Ratio

**Loss Deviation** - A measure of the average deviations of a negative return series; often used as a risk measure. A large downside risk implies that there have been large swings or volatility in the manager's return series when it is below the selected hurdle rate (MAR). Downside risk (also known as downside deviation) attempts to further break down volatility between upside volatility – which is generally favorable since it implies positive outperformance – and downside volatility – which is generally unfavorable and implies loss of capital or returns below an expected or required level.

\[
Loss\ Deviation = \sqrt{\frac{\sum (R_m - MAR)^2}{n}} \times \sqrt{\frac{1}{N_y}}
\]

- \( R_m \) = return for time period when product return is below the selected hurdle rate (MAR)
- \( MAR \) = minimum acceptable return or hurdle rate for each time period
- \( n \) = the number of observations
- \( N_y \) = the number of periods in a year (4 if quarterly data, 12 if monthly data)

See Also:  MAR (Minimum Acceptable Return)

**Excess Returns** - Returns in excess of the risk-free rate, a benchmark or in excess of another manager. A positive excess return indicates that the manager outperformed the benchmark for that period.

Given two return series (typically a manager and a benchmark), \( x_1, \ldots, x_n \) and \( y_1, \ldots, y_n \), the excess return series is defined as \( e_{x1}, \ldots, e_{xn} = x_1 - y_1, \ldots, x_n - y_n \)

\[
Annualized\ Excess\ Return = Annualized\ Manager\ Return - Annualized\ Index\ Return
\]

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Gain Deviation – The Standard Deviation of all positive returns

\[ \text{Gain Deviation} = \sqrt{\frac{\sum (R_m - \text{MAR})^2}{n}} \times \sqrt{(N_y)} \]

\( R_m \) = return for time period when product return is above the selected hurdle rate (MAR)
\( \text{MAR} \) = minimum acceptable return or hurdle rate for each time period
\( n \) = the number of observations
\( N_y \) = the number of periods in a year (4 if quarterly data, 12 if monthly data)

See Also: \( \text{MAR} \) (Minimum Acceptable Return)

Information Ratio - This statistic is computed by subtracting the return of the market from the return of the manager to determine the excess return. The excess return is then divided by the standard deviation of the excess returns (or Tracking Error) to produce the information ratio. This ratio is a measure of the value added per unit of active risk by a manager over an index. Managers taking on higher levels of risk are expected to then generate higher levels of return, so a positive IR would indicate “efficient” use of risk by a manager. This is similar to the Sharpe Ratio, except this calculation is based on excess rates of return versus a benchmark instead of a risk-free rate.

\[ \text{IR} = \frac{\text{Excess Return}}{\text{Tracking Error}} \]

See Also: Excess Return, Tracking Error

Jensen Alpha - The incremental return of a manager over the risk-free rate when the market is stationary. In other words, it is the extra return over the risk-free rate due to non-market factors. This risk-adjusted factor takes into account both the performance of the market as a whole and the volatility of the manager. A positive Jensen Alpha indicates that a manager has produced returns above what would be expected at that risk level, and vice versa for a negative calculation. Jensen Alpha is the Y-intercept of the regression line between all manager and index returns after subtracting the risk-free rate.

\[ \text{Jensen Alpha} = (X - R_f) - \beta(Y - R_f) \]

\( R_f \) = Risk-free rate
\( X \) = the mean return for the manager
\( Y \) = the mean return for the index

See Also: Beta; Alpha

Kurtosis – Kurtosis characterizes the relative peakedness or flatness of a distribution compared with the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution. If there are fewer than four data points, or if the standard deviation of the sample equals zero, Kurtosis returns the N/A error value.

\[ \text{Kurtosis} = \frac{N}{\sum (R_i - M_R)^4} - (3(N-1)^2 + ((N-2)(N-3))) \]

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**MAR (Minimum Acceptable Return)** - A risk/return constraint imposed on the management of a portfolio of assets. The MAR will usually be linked to the nature and level of liabilities that the portfolio is designed to fund. It is often used as a substitute for a normal Capital Market index (i.e. S&P 500 or 91-Day T-Bill) for calculations such as the Sortino Ratio and Downside Risk. It is, as the name would indicate, the annualized return level below which portfolio performance would impact the funding of liabilities or have other negative ramifications.

*See Also: Downside Risk; Sortino Ratio*

**Max Drawdown** - The maximum of the peak-to-trough declines during a specific period. Going sequentially through time with a manager’s cumulative return, it is the “loss” from the highest portfolio value to its lowest point. This is a commonly used hedge fund measure since such funds often employ hedging strategies to protect returns in down markets; hence, the max drawdown is expected to be low.

\[
\text{Max Drawdown} = \text{Max} \left[ \text{ABS} \left( \left( \text{Cumulative Manager Return} - \text{Highwater} \right) / \text{Highwater} \right) \right]
\]

Highwater = The highest cumulative return for the selected time period

*See Also: Return (Cumulative)*

**Max Drawdown Date** – The beginning date of the Max Drawdown Underwater Period.

**Max Drawdown Length** - The longest time period of a decline in price from its highest to lowest point.

**Max Drawdown Period Peak** – The value at which the Max Drawdown Underwater Period began and ended.

*See Also: Max Drawdown Underwater Period*

**Max Drawdown Recovery Length** – The longest period of time from the lowest price point to the previous high-water mark.

**Max Drawdown Underwater Period** – This calculation measures the longest period during which the portfolio was under its high-water mark.

\[
\text{Max Drawdown Underwater Period} = \text{Max Drawdown Length} + \text{Max Drawdown Recovery Length}
\]

**Number of Negative Periods** - The count of negative returns for the selected time frame.

\[
\text{Number of Negative Periods} = \text{Count} \left( R_1 < 0, R_2 < 0 \ldots R_{xi} < 0 \right)
\]

\[ R = \text{the return for each period in decimal format (e.g. 5.20\% = 0.052)} \]

\[ xi = \text{the return of the data series (ith observation)} \]

**Number of Positive Periods** - The count of positive returns, defined as returns greater than or equal to zero, for the selected time frame.

\[
\text{Number of Positive Periods} = \text{Count} \left( R_1 \geq 0, R_2 \geq 0 \ldots R_{xi} \geq 0 \right)
\]
Omega Ratio – The upside probability divided by the downside probability to produce a quality rating for the investment based on all return data in the period.

\[
\Omega(r) = \frac{\int_{r}^{\infty} [1 - F(x)] \, dx}{[\int_{-\infty}^{r} F(x) \, dx]}
\]

F = the cumulative distribution function
r = the threshold defining the gain vs. the loss

Return (Annualized) - The geometric mean of the product’s returns with respect to one year.

\[
\text{Ann Rtn} = \left( (1+R_1) \times \ldots \times (1+R_{n}) \right)^{\left(1/N\right)} - 1
\]

R = the return for each period in decimal format (e.g. 5.20% = 0.052)
N = the number of years

Quarterly conversion of 1/N = 4/n
Monthly conversion of 1/N = 12/n

Return (Cumulative) - The geometric return of the product over time or over a selected period.

\[
\text{Cumulative Rtn} = (1+R_1) \times \ldots \times (1+R_{n}) - 1
\]

R = the return for each period in decimal format (e.g. 5.20% = 0.052)

R-Squared – Otherwise known as the Coefficient of Determination, this statistic, like beta, is a measure of a manager’s movement in relation to the market. Generally, the R-Squared of a manager versus a benchmark is a measure of how closely related the variance of the manager returns and the variance of the benchmark returns are. In other words, the R-Squared measures the percent of a manager’s return patterns that are “explained” by the market and ranges from 0 to 1. For example, an r-squared of 0.90 means that 90% of a portfolio’s return can be explained by movement in the broad market (benchmark).

\[
\text{R-Squared} = (r)^2
\]

r = correlation coefficient

See Also: Correlation Coefficient

Sharpe Ratio - This statistic is computed by subtracting the return of the risk-free index (typically 91-day T-bill or some other cash benchmark) from the return of the manager to determine the risk-adjusted excess return. This excess return is then divided by the standard deviation of the manager. A manager taking on risk, as opposed to investing in cash, is expected to generate higher returns and Sharpe measures how well the manager generated returns with that risk. In other words, it is a measurement of efficiency utilizing the relationship between annualized risk-free return and standard deviation. The higher the Sharpe Ratio, the greater efficiency produced by this manager. For example, a Sharpe Ratio of 1 is better than a ratio of 0.5.

\[
\text{Sharpe} = \frac{\text{Ann Rtn}(x) - \text{Ann Rtn}(R_f)}{\text{Standard Deviation of } x}
\]

R_f = Risk-free rate

See Also: Return (Annualized)
**Skewness** - Skewness characterizes the degree of asymmetry of a distribution around its mean. Positive skewness indicates a distribution with an asymmetric tail extending toward more positive values. Negative skewness indicates a distribution with an asymmetric tail extending toward more negative values.

\[
M_R = \frac{\sum R_I}{N} \\
\text{Skewness} = \frac{N}{(N-1)(N-2)} \left( \sum \frac{(R_i - M_R)(R_i - SD)}{SD} \right)^3
\]

- \( N \) = Number of Periods
- \( R_i \) = Return for period I
- \( M_R \) = Mean of return set R
- \( SD \) = Period Standard Deviation

**Sortino Ratio** - This measure is very similar to the Sharpe Ratio except that it is concerned only with downside volatility (unfavorable) versus total volatility (both favorable, upside volatility and unfavorable, downward volatility). This statistic is computed by subtracting the return of the risk-free index (typically 91-day T-bill or other cash index) from the return of the manager to determine the risk-adjusted excess return. This excess return is then divided by the downside risk of the manager. A manager taking on risk, as opposed to investing in cash, is expected to generate higher returns and Sortino measures how well the manager "spends" that risk, while not penalizing them for upside volatility (outperformance). The higher the Sortino Ratio, the better; a Sortino Ratio of 1 is better than a ratio of 0.5 – higher excess return and/or lower downside risk.

\[
\text{Sortino} = \frac{\text{Ann Rtn}(x) - \text{Ann Rtn}(R_f)}{\text{Downside Risk of } x}
\]

- \( R_f \) = Risk-free rate

See Also: Return (Annualized); Downside Risk; Sharpe Ratio

**Standard Deviation** - A measure of the average deviations of a return series from its mean; often used as a risk measure. A large standard deviation implies that there have been large swings or volatility in the manager’s return series.

\[
\text{StDev}_{(SD)} = \left[ \frac{\sum (x_i - X)^2}{n - 1} \right]^{1/2} \quad \text{or} \quad \text{Square Root of the Variance} = \sqrt{\text{Var}}
\]

\[
\text{Ann StDev} = \text{SD} \times \sqrt{N_y}
\]

- \( x_i \) = the \( i \)th observation
- \( X \) = mean return for series
- \( n \) = the number of observations
- \( N_y \) = the number of periods in a year (4 if quarterly data, 12 if monthly data)

See Also: Variance

**Sterling Ratio** – This is a return/risk ratio. Return (numerator) is defined as the Compound Annualized Rate of Return over the last 3 years. Risk (denominator) is defined as the Average Yearly Maximum Drawdown over the last 3 years less an arbitrary
10%. To calculate this average yearly drawdown, the latest 3 years (36 months) is divided into 3 separate 12-month periods and the maximum drawdown is calculated for each. Then these 3 drawdowns are averaged to produce the Average Yearly Maximum Drawdown for the 3 year period. If three years of data are not available, the available data is used.

**Average Drawdown** = \( \frac{(D_1 + D_2 + D_3)}{3} \)

**Sterling Ratio** = \( \frac{\text{Compound Annualized ROR}}{\text{ABS}((\text{Average Drawdown} - 10\%))} \)

Where \( D_1 \) = Maximum Drawdown for first 12 months
Where \( D_2 \) = Maximum Drawdown for next 12 months
Where \( D_3 \) = Maximum Drawdown for latest 12 months

**Treynor Ratio** - Similar to the Sharpe Ratio, this statistic is computed by subtracting the return of the risk-free index (typically 91-day T-bill or some other cash benchmark) from the return of the manager to determine the risk-adjusted return. This excess return is then divided by the Beta of the portfolio. This is another efficiency ratio that evaluates whether the manager is being rewarded with additional return for each additional unit of risk being taken with risk being defined by Beta, a measure of systematic risk, not Total Risk (standard deviation).

\[
\text{Treynor} = \frac{\text{Ann Rtn}(x) - \text{Ann Rtn}(R_f)}{\beta(x)}
\]

\( R_f \) = Risk-free rate

*See Also: Beta; Return (Annualized); Sharpe Ratio*

**Tracking Error** - A measure of the amount of active risk that is being taken by a manager. This statistic is computed by subtracting the return of a specified benchmark or index from the manager’s return for each period and then calculating the standard deviation of those differences. A higher tracking error indicates a higher level of risk – not necessarily a higher level of return - being taken relative to the specified benchmark. Tracking error only accounts for deviations away from the benchmark, but does not signal in which directions these deviations occur (positive or negative).

\[
\text{TE} = \text{Standard Deviation of Excess Return}
\]

*See Also: Excess Return; Standard Deviation*

**Upside Market Capture Ratio** - A measure of the manager’s performance in up markets relative to the market itself. A value of 110 suggests the manager performs ten percent better than the market when the market is up during the selected time period. The return for the market for each quarter is considered an up market if it is greater than or equal to zero. The Upside Capture Ratio is calculated by dividing the return of the manager during the up market periods by the return of the market for the same period. Generally, the higher the UMC Ratio, the better (If the manager’s UMC Ratio is negative, it means that during that specific time period, the manager’s return for that period was actually negative).

The number of up periods for a given series \((x_1, \ldots, x_n)\) is the number of positive (and zero) returns in the series.

\[
\text{UMC Ratio} = \frac{\{ (1+R_{m1}) \times (1+R_{m1}) \}^{1/N} - 1}{\{ (1+R_{p1}) \times (1+R_{p1}) \}^{1/N} - 1}
\]

\( R_m \) = return for time period when benchmark \((R_y)\) is positive or zero
\( N \) = Number of years (e.g. 6 quarters = 1.5 years; 20 months = 1.667 years)

*See Also: Downside Market Capture Ratio; Up Market Return*
**Up Market Return** - The annualized return for a manager during up markets, defined as periods where the return of the selected benchmark is greater than or equal to zero.

\[
\text{Up Market Return} = \left\{ (1+R_m)^{1/N} \right\} - 1
\]

- \( R_m \) = return for time period when benchmark return is greater than or equal to zero
- \( N \) = Number of years (e.g. 6 quarters = 1.5 years; 20 months = 1.667 years)

**Variance** - A measure of the dispersion of a set of data points around their mean value. It is a mathematical expectation of the average squared deviations from the mean.

\[
\text{Variance} = \frac{\sum (x_i - \bar{X})^2}{n-1} \quad \text{or} \quad \text{Standard Deviation Squared} = (SD)^2
\]

- \( x_i \) = the \( i \)th observation
- \( \bar{X} \) = mean return for series
- \( n \) = the number of observations

*See Also: Standard Deviation*

**Worst Period** - The lowest return for the selected time frame.

\[
\text{Worst Period} = \text{Min} (R_1, R_2...R_x)
\]

- \( R \) = the return for each period in decimal format (e.g. 5.20% = 0.052)
- \( x \) = the return of the data series \((i\)th observation)

*See Also: Best Period*